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Book Review: *Large Scale Dynamics of Interacting Particles*

Large Scale Dynamics of Interacting Particles. H. Spohn, Springer, Berlin, 1991.

Any author who is going to write a mathematical text on a topic in statistical physics, and in particular if he or she wants to treat the problem of deriving the macroscopic behavior of a large system from its microscopic structure, will necessarily have to expose the concept of the two different scales (macro and micro) in space and in time, and his central problem will be to establish theorems transforming that concept into precise mathematical statements. In the concrete case of the present book, it is clear from the title that we will be faced with this type of problem, even more: that the objects of study are systems evolving in time and not equilibrium properties of large systems.

The objects of investigation are "particle systems" which are governed either by a deterministic or a stochastic evolution mechanism. The stochastic systems considered are lattice gases. This has the advantage that by choosing one concrete class of examples, unnecessary repetitions are avoided; it is not felt by the reader as a serious restriction, since the text contains in shorter form plenty of information on other models belonging to the same "universality class." (The reviewer does not believe that the property of being "realistic" is the only criterion for deciding whether systems having a stochastic evolution are legitimate objects of study for a statistical physicist. For him, the study of those systems is justified for another reason: it gives the mathematical physicist a chance to establish in a simplified frame—if one likes: in a toy model—at least partially rigorous results and to work out in full conceptual clarity essential features of the transition from the microscopic to the macroscopic world.) The deterministic systems treated in Spohn's book are classical particles, which means that quantum mechanical models are not taken into consideration—for good reasons: quantum mechanical nonequilibrium theory would be a completely different story. Consequently, the book divides into two parts.

The first part, on classical mechanical particles, covers slightly less than one-half of the total volume. After some introductory paragraphs on many-particle dynamics and equilibrium states, several types of limiting bulk dynamics (i.e., limiting laws for the evolution of the empirical measure in phase space) are discussed: hydrodynamic, Boltzmann–Grad (low-density), mean-field. For all three types in addition to nonequilibrium behavior (laws of large numbers) also equilibrium fluctuations are studied. Finally, the dynamics of a single (test or tracer) particle belonging to a large system is considered. Though a broad range of subjects is covered by this first part, not all of them are treated equally in great detail: the main emphasis is clearly on the section dealing with the Boltzmann–Grad limit. For the mathematician, this section appears to be the hard core of Part I. The reason is clear: the main result one can present in the context chosen by the author is Lanford's[5] derivation of the Boltzmann equation; there exists no result on the hydrodynamic limit for classical particles comparable to Lanford's theorem; finally, mean-field theories are only first steps in the study of many-particle systems, very useful in the years of Kac and McKean, but their value in producing interesting theorems which are able to link macro and microscales in space and time is limited.

The second part, on stochastic systems—more precisely: lattice gases consisting of migrating particles without creation or annihilation of mass, with a time-reversible evolution in equilibrium—is in the reviewer's opinion of greater importance than the first one, because it reflects a more recent development which took place in the years from about 1985 to 1990. There are three topics which play a major role: dynamical equilibrium fluctuations, nonequilibrium dynamics, both for the bulk dynamics, and self-diffusion. In addition, there are sections on "driven" (i.e., nonreversible) lattice gases giving rise to an unviscid Burgers'-type equation in the scaling limit, on phenomena appearing beyond the hydrodynamic time scale (stochastic description of shocks and a discussion of how Navier–Stokes might be understood in the context of lattice gases), and on models which are not precisely lattice gases but share many essential features of them (such as interacting Brownian particles, for example). The main result in equilibrium fluctuation theory besides the corresponding central limit theorem[7] is the Green–Kubo formula and an equivalent variational formula characterizing the bulk diffusion matrix. In nonequilibrium theory the main result presented is (a lattice gas version of) a theorem of Guo, Papanicolaou, and Varadhan[3] which states that the limiting bulk behavior is governed by a nonlinear parabolic equation. In self-diffusion, the central result presented is—in its mathematical essence—a central limit theorem for certain additive functionals of a reversible Markov process.[2,4]

Given the foregoing short list of main theories treated (which is—as I said before—not exhaustive), it is natural to ask if there is a unifying central idea in this monograph or if the book is rather an addition of two more or less independent parts. The reviewer adheres more to the second answer. But he has to say that his judgement is not independent of his personal taste: he prefers the material exposed in Part II and is more likely to use it, whereas Part I is a good source of additional information on Markovian limits for deterministic systems; which means the question of whether there exists such a unifying central idea for the book as a whole is not essential.

If we look at only one part, the same question appears again: is what we see just a collection of various phenomena connected to large-scale limits, or are there unifying notions and methods behind them? For example, in Part II, entropy or entropy production (or, derived from there, the state of minimal entropy production) is such a notion which even leads to a method of proving limit theorems, as shown in ref. 3. Another common feature in the theory of lattice gases exposed here is their reversibility; this indicates that one is on the way to interesting rigorous results in the context of Onsager's reciprocity relations, but for systems which are not just finite-dimensional, but carry a much richer spatial structure. It is also obvious that self-diffusion and bulk diffusion in equilibrium, though conceptually distinct, are intimately connected to each other because they are observed in the same dynamical systems using the same scaling of space and time. In other words, there are indeed some unifying features and notions which connect different sections of the book. On the other hand, the book contains a broad variety of different material, and it is not easy to say which is its predominant character: is it a dissertation on a single subject, treating one question by essentially one method, or is it rather a collection of different approaches to different topics, which are loosely or narrowly related to each other? I think that such an ambiguity is legitimate for a text which is motivated by the aspiration to explain certain *physical* phenomena (discovered a long time before the years 1985–1990) and is not conceived as a monograph on, say, partial differential equations.

Readers who will profit most from Spohn's book are mathematicians or mathematical physicists working with probabilistic methods. But also for more physically minded people on one band and mathematical probabilists on the other it can be of great help, because these readers can use it as an excellent survey of results in a neighboring discipline. In particular, the thoroughly elaborated list of references is of inestimable value equally for specialists and those who only want to get some orientation. For example, a reader might like to learn more in Part I on the results and methods of Bunimovich and Sinai[1] concerning the Lorentz gas; but the available space is not unbounded, and so he or she will appreciate the one-page introduction and the reference on p. 118 even if no proof of

Theorem 8.2 is given. What might be harder for a nonspecialist—say for a person coming from pure probability theory—is to find through this text an access to stimulating problems in mathematical physics, because the author does not offer too much help to overcome the possibly very high language barrier: the book is written to some degree in a kind of insider jargon which, together with a sometimes slightly informal and certainly not pedantic way of dealing with mathematical epsilons (for example, the author says that certain limiting evolutions, which are true in a scaling limit, hold "with probability one") may sound unusual to a person not yet living in the world of probabilistic mathematical physicists.

A final remark. In my opinion, the time of appearance of this monograph is well chosen. There exist by now sufficiently many new results and methods on macroscopic properties of reversible stochastic particle systems, beyond the textbook of Liggett,[6] which makes it desirable to collect and present essential parts of the existing material. On the other hand, the subject is still far from having found its canonical form, it is —fortunately—not yet a scholastic discipline with a well-established system of axioms and conditions, and it contains much more open than solved problems. So Spohn's text will probably be very useful as a guide and reference book to future researchers for some years, which I find is a much nicer role to play than being an encyclopedia of reversible stochastic particle systems for some decades.

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